IRROTATIONAL FLOW OF AN IDEAL FLUID STREAM AROUND A ROTATING PLATE

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Methods of solving problems about nonstationary wing profile motion in an ideal incompressible fluid are elucidated in [1]. Results of investigating certain local singularities of the flow of an ideal fluid stream around a rotating plate are presented in this paper. The magnitude of the suction forces on the plate edges is determined. It is shown that under constant angular and translational velocities only these forces produce drag and lift of the plate. The substantial influence of the location of the point of rotation on the characteristics noted is clarified.

The flow configuration is displayed in Fig. 1, all the dependent and independent quantities are made dimensionless by using the semi-chord of the plate and the unperturbed stream velocity. The x, y coordinate system is coupled to the plate rotating at an angular velocity ω relative to the point $x = x_0$, $y = 0$.

The velocity potential φ of the irrotational flow has the form $\varphi = \varphi_1 + \varphi_2$. Here

$$\varphi_1 = \omega(-(1/4)e^{-2\xi} \sin 2\eta + x_0 e^{-\xi} \sin \eta)$$

is the potential of the irrotational flow caused by plate rotation in a fluid at rest and the orthogonal elliptic coordinates ξ, η are related to the Cartesian by the relationships $x = \cosh \xi \cos \eta$, $y = \sinh \xi \sin \eta$, $0 \le \xi \le \infty$, $-\pi \le \eta \le \pi$. The potential φ_2 is the real part of the complex potential.

$$w_2(z) = -z \cos \alpha - i\sqrt{z^2 - 1} \sin \alpha \; (z = x + iy)$$

of the irrotational flow around the plate at the angle of attack α.

In conformity with φ_1 the tangential velocity component u_1 on the plate is calculated for rotational motion by means of the formula

$$u_1(x) = \pm \omega \frac{2x^2 - 2xx_0 - 1}{2\sqrt{1 - x^2}}.$$

Here and henceforth, the upper and lower signs correspond to the sides of the plate $y = +0$ and $y = -0$. It is seen that the longitudinal velocity at the sharp edges becomes infinite. However, if the rotation occurs with respect to the quarter-chord point $x_0 = -0.5$, then it equals zero independently of the quantity ω at the trailing edge $x = -1$.

Taking account of φ_2 the tangential velocity on the rotating plate is given for a free stream flowing around it by the expression

$$u(x) = -\cos \alpha \pm \frac{\omega\left(2x^2 - 2xx_0 - 1\right) - 2x \sin \alpha}{2\sqrt{1 - x^2}}.$$

Naturally, the Chaplygin–Zhukovskii condition on the sharp edges is not satisfied for $u(x)$ in the general case while the normal velocity component is everywhere finite and determined by the nonpenetration condition.

Let us calculate the suction forces acting on the sharp edges. It follows from the Cauchy–Lagrange integral that not only the unboundedness of the magnitude of the velocity but also the possible unboundedness of the component $\partial\varphi/\partial t$, which is quite substantial in nonstationary flows, can be the case of the infinitely large negative pressures. Taking account of $\partial x/\partial t = y\omega$, $\partial\alpha/\partial t = \omega$, $\partial y/\partial t = (x_0 - x)\omega$, we obtain that for $y = 0$ and $\omega = \text{const}$

$$\partial\varphi_1/\partial t = -\omega^2(x - x_0)^2, \; \partial\varphi_2/\partial t = \omega(x \sin \alpha + \sqrt{1 - x^2}) \cos \alpha.$$

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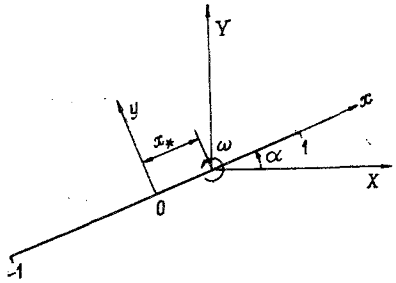

Fig. 1

It is seen that these derivatives are finite at the plate edges $x = \pm 1$; consequently, as in the stationary case the suction force is determined completely by the behavior of the velocity as $x \to \pm 1$.

It is known that if the velocity distribution has the form $V \approx A/\sqrt{l}$ in the neighborhood of a sharp edge (l is the distance from the edge), then the suction force $F_\tau = \pi \rho A^2$ (ρ is the fluid density). We find the coefficient A from the expression presented above for u(x) when $x \to \pm 1$. We have for the coefficient c_τ of a suction force equal to the ratio between F_τ and the velocity head of the unperturbed stream and the plate chord

$$c_\tau(\pm 1) = (\pi/2)[\omega(1/2 \mp x_0) \mp \sin\alpha]^2.$$

Strictly speaking, these formulas are valid for a constant angular velocity of the plate since the contribution of the flow nonstationarity to the magnitude of the pressure is obtained only for this case.

Let us note two circumstances resulting from these formulas: 1) the position of the point of rotation x_0 exerts considerable influence on the quantity c_τ; and 2) the achievement of the identical angle of attack α in its growth and decrease regimes yields substantially different values of the suction force on the sharp edge. The aerodynamic lift and drag coefficients are calculated in terms of the apparent masses, respectively, for constant values of the incoming stream velocity and the angular velocity:

$$c_Y = -\pi\omega(\omega x_0 + \sin\alpha)\sin\alpha, \quad c_X = \pi\omega(\omega x_0 + \sin\alpha)\cos\alpha.$$

It is easy to establish their connection with the quantities c_τ. The coefficient of the total suction force acting on the plate is $\Delta c_\tau = c_\tau(1) - c_\tau(-1) = \pi\omega(\omega x_0 + \sin\alpha)$. If we project Δc_τ onto the X, Y axes then we obtain expressions that agree exactly with expressions for the coefficients c_X, c_Y. This means that in the case under consideration the aerodynamic action on the plate is determined completely by just the suction forces.

In conclusion, let us note that a shift in the point of plate rotation along the normal to it is not reflected in any way in the results obtained.

LITERATURE CITED

1. L. I. Sedov, Plane Problems of Hydrodynamics and Aerodynamics [in Russian], Nauka, Moscow (1966).